GCSE Mathematics (1MA1)

Higher Tier

Scheme of Work

Unit		Title	Estimated hours
1	a	Calculations, checking and rounding	7
	b	Indices, roots, reciprocals and hierarchy of operations	8
	c	Factors, multiples and primes	6
	d	Standard form and surds	6
2	a	Algebra: the basics	8
	b	Setting up, rearranging and solving equations	8
	c	Sequences	6
3	a	Averages and range	7
	b	Representing and interpreting data	8
	c	Scatter graphs	5
4	a	Fractions	8
	b	Percentages	8
	c	Ratio and proportion	8
5	a	Polygons, angles and parallel lines	8
	b	Pythagoras' Theorem and trigonometry	8
6	a	Graphs: the basics and real-life graphs	7
	b	Linear graphs and coordinate geometry	10
	c	Quadratic, cubic and other graphs	8
7	a	Perimeter, area and circles	8
	b	3D forms and volume, cylinders, cones and spheres	8
	c	Accuracy and bounds	6
8	a	Transformations	8
	b	Constructions, loci and bearings	8
9	a	Solving quadratic and simultaneous equations	8
	b	Inequalities	6
10		Probability	10
11		Multiplicative reasoning	8
12		Similarity and congruence in 2D and 3D	8
13	a	Graphs of trigonometric functions	6
	b	Further trigonometry	10
14	a	Collecting data	6
	b	Cumulative frequency, box plots and histograms	7
15		Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics	8
16	a	Circle theorems	7
	b	Circle geometry	6
17		Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof	8
18		Vectors and geometric proof	10
19	a	Reciprocal and exponential graphs; Gradient and area under graphs	8
	b	Direct and inverse proportion	8

UNIT 1: Powers, decimals, HCF and LCM, positive and negative, roots, rounding, reciprocals, standard form, indices and surds

Return to Overview

KEYWORDS

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd, surd, rational, irrational standard form, simplify

1a. Calculations, checking and rounding

- Add, subtract, multiply and divide decimals and whole numbers;
- Multiply or divide by any number between 0 and 1;
- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Use the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways);
- Round to the nearest integer, to a given number of decimal places and to a given number of significant figures;
- Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.

1b. Indices, roots, reciprocals and BIDMAS

- Use index notation including negative powers;
- Recognise powers of 2, 3, 4, 5;
- Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Find the value of calculations using indices including positive, fractional and negative indices;
- Recall that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n as well as, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n;
- Understand that the inverse operation of raising a positive number to a power n is raising the result of this operation to the power $\frac{1}{n}$;
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
- Solve problems using index laws;
- Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
- Use an extended range of calculator functions, including +, −, ×, ÷, x^2, $\sqrt{x}$, memory, x^y, $x^{\frac{1}{y}}$, brackets;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

1c. Factors, multiples and primes

- Identify factors, multiples and prime numbers;
- Find the prime factor decomposition of positive integers – write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
- Solve problems using HCF and LCM, and prime numbers;
- Understand that the prime factor decomposition of a positive integer is unique, whichever factor pair you start with, and that every number can be written as a product of prime factors.

1d. Standard form and surds

- Convert large and small numbers into standard form and vice versa;
- Add and subtract numbers in standard form;
- Multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form;
- Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2;
- Simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$).

UNIT 2: Expressions, substituting into simple formulae, expanding and factorising, equations, sequences and inequalities, simple proof

KEYWORDS

Expression, identity, equation, formula, substitute, term, 'like' terms, index, power, negative and fractional indices, collect, substitute, expand, bracket, factor, factorise, quadratic, linear, simplify, approximate, arithmetic, geometric, function, sequence, nth term, derive

2a. Algebra: the basics

- Write an expression;
- Know the difference between a term, expression, equation, formula and an identity;
- Manipulate an expression by collecting like terms;
- Substitute positive and negative numbers into expressions such as $3x + 4$ and $2x^3$ and then into expressions involving brackets and powers;
- Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g. $l \times w$, $v = u + at$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$
- Use instances of index laws for positive integer powers;
- Use index notation (positive powers) when multiplying or dividing algebraic terms;
- Use instances of index laws, including use of zero, fractional and negative powers;
- Multiply a single term over a bracket;
- Recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;

2b. Setting up, rearranging and solving equations

- Set up simple equations from word problems and derive simple formulae;
- Understand the ≠ symbol (not equal), e.g. $6x + 4 \neq 3(x + 2)$, and introduce identity ≡ sign;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Set up and solve linear equations to solve to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Substitute positive and negative numbers into a formula, solve the resulting equation including brackets, powers or standard form;
- Use and substitute formulae from mathematics and other subjects, including the kinematics formulae $v = u + at$, $v^2 - u^2 = 2as$, and $s = ut + \frac{1}{2}at^2$;
- Change the subject of a simple formula, i.e. linear one-step, such as $x = 4y$;
- Change the subject of a formula, including cases where the subject is on both sides of the original formula, or involving fractions and small powers of the subject;
- Simple proofs and use of ≡ in "show that" style questions; know the difference between an equation and an identity;
- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations.

2c. Sequences

- Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences;
- Generate sequences of numbers, squared integers and sequences derived from diagrams;
- Describe in words a term-to-term sequence and identify which terms cannot be in a sequence;
- Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
- Find and use (to generate terms) the *n*th term of an arithmetic sequence;
- Use the *n*th term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number;
- Identify which terms cannot be in a sequence by finding the *n*th term;
- Continue a quadratic sequence and use the *n*th term to generate terms;
- Find the nth term of quadratic sequences;
- Distinguish between arithmetic and geometric sequences;
- Use finite/infinite and ascending/descending to describe sequences;
- Recognise and use simple geometric progressions (rn where n is an integer, and r is a rational number > 0 or a surd);
- Continue geometric progression and find term to term rule, including negative, fraction and decimal terms;
- Solve problems involving sequences from real life situations.

Unit 3: Averages and Range, collecting data, representing data

3a. Averages and range

KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, sample, population, stem and leaf, frequency, table, estimate

- Design and use two-way tables for discrete and grouped data;
- Use information provided to complete a two-way table;
- Sort, classify and tabulate data and discrete or continuous quantitative data;
- Calculate mean and range, find median and mode from small data set;
- Use a spreadsheet to calculate mean and range, and find median and mode;
- Recognise the advantages and disadvantages between measures of average;
- Construct and interpret stem and leaf diagrams (including back-to-back diagrams):
- find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- Calculate the mean, mode, median and range from a frequency table (discrete data);
- Construct and interpret grouped frequency tables for continuous data:
- for grouped data, find the interval which contains the median and the modal class;
- estimate the mean with grouped data;
- understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values.

3b. Representing and interpreting data

KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, stem and leaf, frequency, table, sort, pie chart, estimate

- Know which charts to use for different types of data sets;
- Produce and interpret composite bar charts;
- Produce and interpret comparative and dual bar charts;
- Produce and interpret pie charts:
- find the mode and the frequency represented by each sector;
- compare data from pie charts that represent different-sized samples;
- Produce and interpret frequency polygons for grouped data:
- from frequency polygons, read off frequency values, compare distributions, calculate total population, mean, estimate greatest and least possible values (and range);
- Produce frequency diagrams for grouped discrete data:
- read off frequency values, calculate total population, find greatest and least values;
- Produce line graphs:
- read off frequency values, calculate total population, find greatest and least values;
- Construct and interpret time–series graphs, comment on trends;
- Compare the mean and range of two distributions, or median or mode as appropriate;
- Recognise simple patterns, characteristics relationships in bar charts, line graphs and frequency polygons.

3c. Scatter graphs

KEYWORDS

scatter graph, line of best fit, correlation, positive, negative, anomaly, outlier

- Draw and interpret scatter graphs;
- Interpret scatter graphs in terms of the relationship between two variables;
- Draw lines of best fit by eye, understanding what these represent;
- Identify outliers and ignore them on scatter graphs;
- Use a line of best fit, or otherwise, to predict values of a variable given values of the other variable;
- Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem;
- Understand that correlation does not imply causality, and appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship' but merely 'no linear correlation';
- Explain an isolated point on a scatter graph;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

UNIT 4: Fractions, percentages, ratio and proportion

KEYWORDS

Addition, subtraction, multiplication, division, fractions, mixed, improper, recurring, reciprocal, integer, decimal, termination, percentage, VAT, increase, decrease, multiplier, profit, loss, ratio, proportion, share, parts

4a. Fractions

- Express a given number as a fraction of another;
- Find equivalent fractions and compare the size of fractions;
- Write a fraction in its simplest form, including using it to simplify a calculation, e.g. $50 \div 20 = \frac{50}{20} = \frac{5}{2} = 2.5$;
- Find a fraction of a quantity or measurement, including within a context;
- Convert a fraction to a decimal to make a calculation easier;
- Convert between mixed numbers and improper fractions;
- Add, subtract, multiply and divide fractions;
- Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;
- Add and subtract fractions, including mixed numbers;
- Understand and use unit fractions as multiplicative inverses;
- By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
- Convert a fraction to a recurring decimal;
- Convert a recurring decimal to a fraction;
- Find the reciprocal of an integer, decimal or fraction.

4b. Percentages

- Convert between fractions, decimals and percentages;
- Express a given number as a percentage of another number;
- Express one quantity as a percentage of another where the percentage is greater than 100%
- Find a percentage of a quantity;
- Find the new amount after a percentage increase or decrease;
- Work out a percentage increase or decrease, including: simple interest, income tax calculations, value of profit or loss, percentage profit or loss;
- Compare two quantities using percentages, including a range of calculations and contexts such as those involving time or money;
- Find a percentage of a quantity using a multiplier;
- Use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Find the original amount given the final amount after a percentage increase or decrease (reverse percentages), including VAT;
- Use calculators for reverse percentage calculations by doing an appropriate division;
- Use percentages in real-life situations, including percentages greater than 100%;
- Describe percentage increase/decrease with fractions, e.g. 150% increase means $2\frac{1}{2}$ times as big;
- Understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents, and choose fractions, decimals or percentages appropriately for calculations.

4c. Ratio and proportion

- Express the division of a quantity into a number parts as a ratio;
- Write ratios in form $1 : m$ or $m : 1$ and to describe a situation;
- Write ratios in their simplest form, including three-part ratios;
- Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
- Use a ratio to find one quantity when the other is known;
- Write a ratio as a fraction;
- Write a ratio as a linear function;
- Identify direct proportion from a table of values, by comparing ratios of values;
- Use a ratio to compare a scale model to real-life object;
- Use a ratio to convert between measures and currencies, e.g. £1.00 = €1.36;
- Scale up recipes;
- Convert between currencies.

UNIT 5: Angles, polygons, parallel lines; Right-angled triangles: Pythagoras and trigonometry

KEYWORDS

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, Pythagoras' Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, segment, length

5a. Polygons, angles and parallel lines

- Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
- Understand 'regular' and 'irregular' as applied to polygons;
- Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle;
- Use symmetry property of an isosceles triangle to show that base angles are equal;
- Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Explain why the angle sum of a quadrilateral is 360°;
- Understand and use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is 360°;
- Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
- Use the angle sums of irregular polygons;
- Calculate and use the sums of the interior angles of polygons, use the sum of angles in a triangle to deduce and use the angle sum in any polygon and to derive the properties of regular polygons;
- Use the sum of the exterior angles of any polygon is 360°;
- Use the sum of the interior angles of an n-sided polygon;
- Use the sum of the interior angle and the exterior angle is 180°;
- Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons;
- Calculate the angles of regular polygons and use these to solve problems;
- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
- Use angle facts to demonstrate how shapes would 'fit together', and work out interior angles of shapes in a pattern.

5b. Pythagoras' Theorem and trigonometry

- Understand, recall and use Pythagoras' Theorem in 2D;
- Given three sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
- Find the length of a shorter side in a right-angled triangle;
- Calculate the length of a line segment AB given pairs of points;
- Give an answer to the use of Pythagoras' Theorem in surd form;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

UNIT 6: Real-life and algebraic linear graphs, quadratic and cubic graphs, the equation of a circle, plus rates of change and area under graphs made from straight lines

KEYWORDS

Coordinate, axes, 3D, Pythagoras, graph, speed, distance, time, velocity, quadratic, solution, root, function, linear, circle, cubic, approximate, gradient, perpendicular, parallel, equation

6a. Graphs: the basics and real-life graphs

- Identify and plot points in all four quadrants;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
- Find the coordinates of the midpoint of a line segment from coordinates;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.
- Find the equation of the line through two given points.

6b. Linear graphs and coordinate geometry

- Plot and draw graphs of $y = a$, $x = a$, $y = x$ and $y = -x$, drawing and recognising lines parallel to axes, plus $y = x$ and $y = -x$;
- Identify and interpret the gradient of a line segment;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Identify and interpret the gradient and y-intercept of a linear graph given by equations of the form $y = mx + c$;
- Find the equation of a straight line from a graph in the form $y = mx + c$;
- Plot and draw graphs of straight lines of the form $y = mx + c$ with and without a table of values;
- Sketch a graph of a linear function, using the gradient and y-intercept (i.e. without a table of values);
- Find the equation of the line through one point with a given gradient;
- Identify and interpret gradient from an equation $ax + by = c$;
- Find the equation of a straight line from a graph in the form $ax + by = c$;
- Plot and draw graphs of straight lines in the form $ax + by = c$;
- Interpret and analyse information presented in a range of linear graphs:
- use gradients to interpret how one variable changes in relation to another;
- find approximate solutions to a linear equation from a graph;
- identify direct proportion from a graph;
- find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Select and use the fact that when $y = mx + c$ is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$.

6c. Quadratics: Factorising and Graphs; Cubic Graphs and other graphs

- Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to $(2x + 3y)(3x - y)$;
- Know that squaring a linear expression is the same as expanding double brackets;
- Factorise quadratic expressions of the form ax2 + bx + c;
- Factorise quadratic expressions using the difference of two squares.
- Solve quadratic equations by factorisation and completing the square;
- Solve quadratic equations that need rearranging;
- Set up and solve quadratic equations;
- Solve quadratic equations by using the quadratic formula;
- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots and y-intercept, turning point;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph;
- Interpret graphs of quadratic functions from real-life problems;
- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;

- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;

UNIT 7: Perimeter, area and volume, plane shapes and prisms, circles, cylinders, spheres, cones; Accuracy and bounds

KEYWORDS

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, nets, isometric, symmetry, vertices, edge, face, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, composite, sphere, cone, capacity, hemisphere, segment, frustum, bounds, accuracy, surface area

7a. Perimeter, area and circles

- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the area of compound shapes made from triangles, rectangles, trapezia and parallelograms using a variety of metric measures;
- Find the perimeter of a rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the perimeter of compound shapes made from triangles and rectangles;
- Estimate area and perimeter by rounding measurements to 1 significant figure to check reasonableness of answers.
- Recall the definition of a circle and name and draw parts of a circle;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference = 2πr = πd and area of a circle = πr2) using a variety of metric measures;
- Use π ≈ 3.142 or use the π button on a calculator;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
- Calculate arc lengths, angles and areas of sectors of circles;
- Find radius or diameter, given area or circumference of circles in a variety of metric measures;
- Give answers in terms of π;
- Form equations involving more complex shapes and solve these equations.

Ensure that students know it is more accurate to leave answers in terms of π, but only when asked to do so.

7b. 3D forms and volume, cylinders, cones and spheres

- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes with and without a diagram;
- Draw sketches of 3D solids;
- Identify planes of symmetry of 3D solids, and sketch planes of symmetry;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;
- Convert between metric volume measures;
- Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm3;
- Use volume to solve problems;
- Estimating surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers.
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Find the volume and surface area of a cylinder;
- Recall and use the formula for volume of pyramid;
- Find the surface area of a pyramid;
- Use the formulae for volume and surface area of spheres and cones;
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
- Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
- Give answers in terms of π;
- Form equations involving more complex shapes and solve these equations.

7c. Accuracy and bounds

- Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
- Calculate the upper and lower bounds of an expression involving the four operations;
- Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
- Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
- Calculate the upper and lower bounds of calculations, particularly when working with measurements;
- Use inequality notation to specify an error bound.

UNIT 8: Transformations; Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

KEYWORDS

Rotation, reflection, translation, transformation, enlargement, scale factor, vector, centre, angle, direction, mirror line, centre of enlargement, describe, distance, congruence, similar, combinations, single, corresponding, constructions, compasses, protractor, bisector, bisect, line segment, perpendicular, loci, bearing

8a. Transformations

- Distinguish properties that are preserved under particular transformations;
- Recognise and describe rotations – know that that they are specified by a centre and an angle;
- Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
- Identify the equation of a line of symmetry;
- Recognise and describe reflections on a coordinate grid – know to include the mirror line as a simple algebraic equation, $x = a$, $y = a$, $y = x$, $y = -x$ and lines not parallel to the axes;
- Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also $y = x$ and $y = -x$;
- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
- Enlarge a shape on a grid without a centre specified;
- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
- Know that an enlargement on a grid is specified by a centre and a scale factor;
- Identify the scale factor of an enlargement of a shape;
- Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing;
- Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

8b. Constructions, loci and bearings

- Draw 3D shapes using isometric grids;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
- Use the standard ruler and compass constructions:
- bisect a given angle;
- construct a perpendicular to a given line from/at a given point;
- construct angles of 90°, 45°;
- perpendicular bisector of a line segment;
- Construct:
- a region bounded by a circle and an intersecting line;
- a given distance from a point and a given distance from a line;
- equal distances from two points or two line segments;
- regions which may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci, including in 3D;
- Use constructions to solve loci problems including with bearings;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line.

UNIT 9: Algebra: Solving quadratic equations and inequalities, solving simultaneous equations algebraically

KEYWORDS

Quadratic, solution, root, linear, solve, simultaneous, inequality, completing the square, factorise, rearrange, surd, function, solve, circle, sets, union, intersection

9a. Solving simultaneous equations

- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
    - linear / linear, including where both need multiplying;
    - linear / quadratic;
- Set up and solve a pair of simultaneous equations in two variables for each of the above scenarios, including to represent a situation;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points and use to get approximate solutions to simultaneous equations;
- Interpret the solution in the context of the problem;
- solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;

9b. Inequalities

- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Solve two linear inequalities in x, find the solution sets and compare them to see which value of x satisfies both solve linear inequalities in two variables algebraically;
- Use the correct notation to show inclusive and exclusive inequalities.
- Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;
- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation;
- for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of $x^2 - 3x - 10 < 0$ as $\{x: -3 < x < 5\}$;
- Solve linear inequalities in two variables graphically;

UNIT 10: Probability

KEYWORDS

Probability, mutually exclusive, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, Venn diagram, fairness, experimental

- Write probabilities using fractions, percentages or decimals;
- Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Find the probability of successive events, such as several throws of a single dice;
- List all outcomes for single events, and combined events, systematically;
- Draw sample space diagrams and use them for adding simple probabilities;
- Know that the sum of the probabilities of all outcomes is 1;
- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Find a missing probability from a list or two-way table, including algebraic terms;
- Understand conditional probabilities and decide if two events are independent;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Understand selection with or without replacement;
- Calculate the probability of independent and dependent combined events;
- Use a two-way table to calculate conditional probability;
- Use a tree diagram to calculate conditional probability;
- Use a Venn diagram to calculate conditional probability;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

UNIT 11: Multiplicative reasoning: direct and inverse proportion, relating to graph form for direct, compound measures, repeated proportional change

KEYWORDS

Ratio, proportion, best value, unitary, proportional change, compound measure, density, mass, volume, speed, distance, time, density, mass, volume, pressure, acceleration, velocity, inverse, direct, constant of proportionality

- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when A:B are in the ratio 3:5, A is $\frac{3}{5}$ B. When 4a = 7b, then a = $\frac{7b}{4}$ or a:b is 7:4;
- Solve proportion problems using the unitary method;
- Work out which product offers best value and consider rates of pay;
- Work out the multiplier for repeated proportional change as a single decimal number;
- Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation;
- Understand and use compound measures and:
- convert between metric speed measures;
- convert between density measures;
- convert between pressure measures;
- Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question);
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
- Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of k in $y = kx$;
- Set up and use equations to solve word and other problems involving direct proportion (this is covered in more detail in unit 19);
- Relate algebraic solutions to graphical representation of the equations;
- Recognise when values are in inverse proportion by reference to the graph form;
- Set up and use equations to solve word and other problems involving inverse proportion, and relate algebraic solutions to graphical representation of the equations.

UNIT 12: Similarity and congruence in 2D and 3D

KEYWORDS

Congruence, side, angle, compass, construction, shape, volume, length, area, volume, scale factor, enlargement, similar, perimeter, frustum

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
- Solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
- Use formal geometric proof for the similarity of two given triangles;
- Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
- Find missing lengths, areas and volumes in similar 3D solids;
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;
- Use the relationship between enlargement and areas and volumes of simple shapes and solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.
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UNIT 13: Sine and cosine rules, $\frac{1}{2}ab$ sin C, trigonometry and Pythagoras' Theorem in 3D, trigonometric graphs, and accuracy and bounds

KEYWORDS

Axes, coordinates, sine, cosine, tan, angle, graph, transformations, side, angle, inverse, square root, 2D, 3D, diagonal, plane, cuboid

13a. Graphs of trigonometric functions (A8, A12, A13, G21)	**Teaching time** 5–7 hours

- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) y = sin x, y = cos x and y = tan x for angles of any size.
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45° , 60° and 90° and exact value of tan θ for θ = 0°, 30°, 45° and 60° and find them from graphs.
- Apply to the graph of y = f(x) the transformations y = –f(x), y = f(–x) for sine, cosine and tan functions f(x).
- Apply to the graph of y = f(x) the transformations y = f(x) + a, y = f(x + a) for sine, cosine and tan functions f(x).

13b. Further trigonometry	**Teaching time**
(N16, G11, G20, G22, G23)	9–11 hours

- Know and apply Area = $\frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle.
- Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
- Use the sine and cosine rules to solve 3D problems.
- Understand the language of planes, and recognise the diagonals of a cuboid.
- Solve geometrical problems on coordinate axes.
- Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
- Calculate the length of a diagonal of a cuboid.
- Find the angle between a line and a plane.

UNIT 14: Statistics and sampling, cumulative frequency and histograms

KEYWORDS

Sample, population, fraction, decimal, percentage, bias, stratified sample, random, cumulative frequency, box plot, histogram, frequency density, frequency, mean, median, mode, range, lower quartile, upper quartile, interquartile range, spread, comparison, outlier

14a. Collecting data	**Teaching time**
(S1)	5–7 hours

- Specify the problem and plan:
- decide what data to collect and what analysis is needed;
- understand primary and secondary data sources;
- consider fairness;
- Understand what is meant by a sample and a population;
- Understand how different sample sizes may affect the reliability of conclusions drawn;
- Identify possible sources of bias and plan to minimise it;
- Write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative (see note)

14b. Cumulative frequency, box plots and histograms

- Use statistics found in all graphs/charts in this unit to describe a population;
- Know the appropriate uses of cumulative frequency diagrams;
- Construct and interpret cumulative frequency tables;
- Construct and interpret cumulative frequency graphs/diagrams and from the graph:
- estimate frequency greater/less than a given value;
- find the median and quartile values and interquartile range;
- Compare the mean and range of two distributions, or median and interquartile range, as appropriate;
- Interpret box plots to find median, quartiles, range and interquartile range and draw conclusions;
- Produce box plots from raw data and when given quartiles, median and identify any outliers;
- Know the appropriate uses of histograms;
- Construct and interpret histograms from class intervals with unequal width;
- Use and understand frequency density;
- From histograms:
  - complete a grouped frequency table;
  - understand and define frequency density;
  - Estimate the mean from a histogram;
  - Estimate the median from a histogram with unequal class widths or any other information from a histogram, such as the number of people in a given interval.

UNIT 15: Cubics and other more complex Graphs: Expanding more than two brackets, sketching graphs, graphs of circles, cubics and reciprocals

KEYWORDS

Sketch, estimate, quadratic, cubic, function, factorising, simultaneous equation, graphical, algebraic

- Expand the product of more than two linear expressions;
- Sketch graphs of simple cubic functions, given as three linear expressions;
- Draw graphs of simple cubic functions using tables of values;
- Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Draw graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$ using tables of values;
- Draw circles, centre the origin, equation $x^2 + y^2 = r^2$.
- Solve simultaneous equations graphically:
  - find graphically the intersection points of a given straight line with a circle;
  - find graphically the intersection points of a other graphs eg quadratics, circles, trig
- Show the solution set of several inequalities in two variables on a graph;
- Use iteration with simple converging sequences.

UNIT 16: Circle theorems and circle geometry

KEYWORDS

Radius, centre, tangent, circumference, diameter, gradient, perpendicular, reciprocal, coordinate, equation, substitution, chord, triangle, isosceles, angles, degrees, cyclic quadrilateral, alternate, segment, semicircle, arc, theorem

16a. Circle theorems (G9, G10) — Teaching time 6–8 hours

- Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
- Prove and use the facts that:
- the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
- the angle in a semicircle is a right angle;
- the perpendicular from the centre of a circle to a chord bisects the chord;
- angles in the same segment are equal;
- alternate segment theorem;
- opposite angles of a cyclic quadrilateral sum to 180°;
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
- Find and give reasons for missing angles on diagrams using:
- circle theorems;
- isosceles triangles (radius properties) in circles;
- the fact that the angle between a tangent and radius is 90°;
- the fact that tangents from an external point are equal in length.

16b. Circle geometry

- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
- Find the equation of a tangent to a circle at a given point, by:
- finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
- finding the gradient of the tangent perpendicular to it;
- using the given point;
- Recognise and construct the graph of a circle using $x^2 + y^2 = r^2$ for radius r centred at the origin of coordinates.

UNIT 17: Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof

KEYWORDS

Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate

- Rationalise the denominator involving surds;
- Simplify algebraic fractions;
- Multiply and divide algebraic fractions;
- Solve quadratic equations arising from algebraic fraction equations;
- Change the subject of a formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears;
- Change the subject of a formula such as $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where all variables are in the denominators;
- Solve 'Show that' and proof questions using consecutive integers (n, $n + 1$), squares a^2, b^2, even numbers $2n$, odd numbers $2n + 1$;
- Use function notation;
- Find $f(x) + g(x)$ and $f(x) - g(x)$, $2f(x)$, $f(3x)$ etc algebraically;
- Find the inverse of a linear function;
- Know that $f^{-1}(x)$ refers to the inverse function;
- For two functions $f(x)$ and $g(x)$, find $gf(x)$.

UNIT 18: Vectors and geometric proof

Teaching time 9–11 hours

Return to Overview

KEYWORDS

Vector, direction, magnitude, scalar, multiple, parallel, collinear, proof, ratio, column vector

- Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
- Understand that $2\mathbf{a}$ is parallel to $\mathbf{a}$ and twice its length, and that $\mathbf{a}$ is parallel to $-\mathbf{a}$ in the opposite direction.
- Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
- Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
- Find the length of a vector using Pythagoras' Theorem.
- Calculate the resultant of two vectors.
- Solve geometric problems in 2D where vectors are divided in a given ratio.
- Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

UNIT 19: Direct and indirect proportion: using statements of proportionality, reciprocal and exponential graphs, rates of change in graphs, functions, transformations of graphs

KEYWORDS

Reciprocal, linear, gradient, quadratic, exponential, functions, direct, indirect, proportion, estimate, area, rate of change, distance, time, velocity, transformations, cubic, transformation, constant of proportionality

19a. Reciprocal and exponential graphs; Gradient and area under graphs

Teaching time 7–9 hours

(R14, R15, A7, A12, A13, A14, A15)

- Recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$
- State the value of x for which the equation is not defined;
- Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k and integer values of x;
- Use calculators to explore exponential growth and decay;
- Set up, solve and interpret the answers in growth and decay problems;
- Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of f(x) + a, or f(x – a):
- apply to the graph of y = f(x) the transformations y = –f(x), y = f(–x) for linear, quadratic, cubic functions;
- apply to the graph of y = f(x) the transformations y = f(x) + a, y = f(x + a) for linear, quadratic, cubic functions;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
- for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
- for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying;
- Interpret the rate of change of unit price in price graphs.

19b. Direct and inverse proportion

- Recognise and interpret graphs showing direct and indirect proportion;
- Identify direct proportion from a table of values, by comparing ratios of values, for x squared and x cubed relationships;
- Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
- Set up and use equations to solve word and other problems involving direct proportion;
- Use $y = kx$ to solve direct proportion problems, including questions where students find k, and then use k to find another value;

- Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
- Solve problems involving inverse proportionality;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.